AVIRAL
JEE | NEET | FOUNDATIONS

WEEKLY TEST TARGET - JEE - TEST - 15
SOLUTION Date 18-08-2019

[PHYSICS]

1. $\text{Stress} = \frac{\text{Force}}{\text{area}}$.

In the present case, force applied and area of cross-section of wires are same, therefore stress has to be the same.

$$\text{Strain} = \frac{\text{Stress}}{Y}$$

Since the Young's modulus of steel wire is greater than the copper wire, therefore, strain in case of steel wire is less than that in case of copper wire.

2. When a wire is suspended from the ceiling and stretched under the action of a weight (F) suspended from its other end, the force exerted by the ceiling on it is equal and opposite to the weight. However, the tension at any cross-section A of the wire is just F and not $2F$. Hence, tensile stress which is equal to the tension per unit area is equal to F/A.

3. $l = \frac{FL}{AY} \therefore l \propto \frac{1}{r^2}$ (F, L and Y are constant)

$$\frac{l_1}{l_2} = \left(\frac{r_2}{r_1}\right)^2 = (2)^2 = 4$$

4. If coefficient of volume expansion is α and rise in temperature is $\Delta\theta$ then $\Delta V = V\alpha\Delta\theta \Rightarrow \frac{\Delta V}{V} = \alpha\Delta\theta$

Volume elasticity $\beta = \frac{P}{\Delta V/V} = \frac{P}{\alpha\Delta\theta} \Rightarrow \Delta\theta = \frac{P}{\alpha\beta}$

5. $\eta = \frac{Fl}{A\Delta l} = \frac{Fl}{l^2\Delta l} = \frac{F}{l\Delta l}$ or $\Delta l \propto \frac{1}{l}$

If ℓ is halved, then $\Delta\ell$ is doubled.

6. $B = -\dfrac{\Delta P}{\Delta V/V} = -\dfrac{V\Delta P}{\Delta V}$

$$= -\frac{1.5 \times 140 \times 10^3}{-0.2 \times 10^{-3}} = 1.05 \times 10^9 \text{ Pa}$$

7. Work done in stretching a wire .

$$W = \frac{1}{2}Fl = \frac{1}{2} \times 10 \times 0.5 \times 10^{-3} = 2.5 \times 10^{-3} \text{ J}$$

Work done to displace it through 1.5 mm

$$W = F \times l = 5 \times 10^{-3} \text{ J}$$

The ratio of above two work = 1 : 2

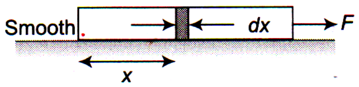
8. At extension l_1, the stored energy $= \frac{1}{2}Kl_1^2$

At extension l_2, the stored energy $= \frac{1}{2}Kl_2^2$

Work done in increasing its extension from l_1 to l_2

$$= \frac{1}{2}K(l_2^2 - l_1^2)$$

9. Tension, $T = \dfrac{F}{L_0} \cdot x\, x$



Stress, $\sigma = \dfrac{T}{A} = \dfrac{F}{AL_0}x$

$$dU = \frac{1}{2} \cdot \frac{\sigma^2}{Y} Adx = \frac{1}{2} \frac{F^2}{A^2 L_0^2} \cdot x^2 \frac{A}{Y} dx$$

or $$dU = \frac{F^2}{2A^2 L_0^2 Y} \cdot x^2 dx$$

$\Rightarrow$ $$U = \frac{F^2}{2AYL_0^2} \int_0^{L_0} x^2 dx$$

$$U = \frac{F^2}{2AYL_0^2} \cdot \frac{L_0^3}{3} = \frac{F^2 L_0}{6AY}$$

10. According to Hooke's law

Within the elastic limit, stress is directly proportional to the strain i.e., Stress $\propto$ Strain

or Stress = k strain

$$\frac{\text{Stress}}{\text{Strain}} = k$$

when k is the proportionality constant and is known as modulus of elasticity.

11. Here, $r = 10\text{ mm} = 10 \times 10^{-3} = 10^{-3}$ m

$L = 1$ m, $F = 100$ kN $= 100 \times 10^{3}$ N $= 10^{5}$ N

Stress produced in the rod is

$$\text{Strain} = \frac{F}{A} = \frac{F}{\pi r^2} = \frac{100 \times 10^3\text{ N}}{3.14 \times (10^{-2}\text{ m})^2}$$

$$= 3.18 \times 10^8 \text{ N m}^{-2}$$

12. Young's modulus depends upon the nature of material and not the radii of the wires.

13. $Y = \dfrac{Fl}{A\Delta l}$ or $F = \dfrac{YA\Delta l}{l}$

or $F = \dfrac{2.2 \times 10^{11} \times 2 \times 10^{-6} \times 0.5 \times 10^{-3}}{2} = 1.1 \times 10^2\text{N}$

14. D

15. $U = \dfrac{1}{2}\left(\dfrac{YA}{L}\right)l^2 \therefore U \propto l^2$

$$\frac{U_2}{U_1} = \left(\frac{l_2}{l_1}\right)^2 = \left(\frac{10}{2}\right)^2 = 25 \Rightarrow U_2 = 25U_1$$

i.e., potential energy of the spring will be 25 V

16. $W = \dfrac{1}{2}Fl \therefore W \propto l$ (F is constant)

$$\therefore \quad \frac{W_1}{W_2} = \frac{l_1}{l_2} = \frac{l}{2l} = \frac{1}{2}$$

17. $U = \dfrac{1}{2} \times \dfrac{YAl^2}{L} = \dfrac{1}{2} \times \dfrac{2 \times 10^{11} \times 3 \times 10^{-6} \times (1 \times 10^{-3})^2}{4}$

$= 0.075$ J

18. The elastic pot. Energy $= \dfrac{1}{2}\text{stress} \times \text{strain}.$

$$= \frac{1}{2}Y(\text{strain})^2 = \frac{1}{2}Y\left(\frac{\Delta l}{L}\right)^2$$

$$\therefore \quad \frac{U_2}{U_1} \propto \left(\frac{\Delta l_2}{\Delta l_1}\right)^2 = \left(\frac{10}{2}\right)^2$$

$$\frac{U_2}{U_1} = 25$$

$$\Rightarrow \quad U_2 = 25U_1$$

So the correct choice is (b).

19. Orbital velocity of the satellite is

$v=\sqrt{\frac{GM_E}{r}}$ where M_E is the mass of the earth

Kinetic energy, $K=\frac{1}{2}mv^2=\frac{GM_E m}{2r}$

where m is the mass of the satellite.

$$K \propto \frac{1}{r}$$

Hence, option (b) in incorrect.

Linear momentum, $p=mv=m\sqrt{\frac{GM_E}{r}}$

$$p \propto \frac{1}{\sqrt{r}}$$

Hence, option (c) is incorrect.

Frequency of revolution, $\nu=\frac{1}{T}=\frac{1}{2\pi}\sqrt{\frac{GM_E}{r^3}}$

$$\nu \propto \frac{1}{r^{3/2}}$$

Hence, option (d) is correct.

20. Time period, $T=\frac{2\pi R}{\sqrt{\frac{GM_m}{R}}}=\frac{2\pi R^{3/2}}{\sqrt{GMm}}$

where the symbols have their meanings as given. Squaring both sides, we get

$$T^2=\frac{4\pi^2 R^3}{GMm}$$

21. Total energy of the orbiting satellite of mass m having orbital radius r is

$E=-\frac{GMm}{2r}$ where M is the mass of the planet.

Additional kinetic energy required to transfer the satellite from a circular orbit of radius R_1 to another radius R_2 is

$$=E_2-E_1$$

$$=-\frac{GMm}{2R_E}-\left(-\frac{GMm}{2R_1}\right)=-\frac{GMm}{2R_2}+\frac{GMm}{2R_1}$$

$$=\frac{GMm}{2}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

22. Total energy of orbiting satellite at a height h is

$$E = -\frac{GM_E m}{2(R_E + h)}$$

The total energy of the satellite at infinity is zero.

$\therefore$ Energy expended to rocket the satellite out of the earth's gravitational field is

$$\Delta E = E_\infty - E$$

$$= 0 - \left(-\frac{GM_E m}{2(R_E + h)}\right) = \frac{GM_E m}{2(R_E + h)}$$

23. Let m_1 is mass of core and m_2 is of outer portion

$$m_1 = \frac{4}{3}\pi R^3 \rho_1, \quad m_2 = \frac{4}{3}\pi[(2R)^3 - R^3]\rho_2$$

Given that: $\frac{Gm_1}{R^2} = \frac{G(m_1 + m_2)}{(2R)^2} \Rightarrow \frac{\rho_1}{\rho_2} = \frac{7}{3}$

24. $v_1 r_1 = v_2 r_2$

$$\frac{KE_1}{KE_2} = \frac{\frac{1}{2}mv_1^2}{\frac{1}{2}mv_2^2} = \left(\frac{v_1}{v_2}\right)^2 = \left(\frac{r_2}{r_1}\right)^2$$

25 For two points on same orbit $L = mv_A r_A = mvr_B$

$$v_A = \frac{vr_B}{r_A} \qquad \text{(i)}$$

For two points on different orbits.

$$v = \sqrt{\frac{GM}{r}}\ \frac{v_0}{v_A} = \left(\frac{r_A}{1.2r_A}\right)^{1/2}$$

$$v_0 = v_A\left(\frac{r_A}{1.2r_A}\right)^{1/2} = \frac{vr_B}{r_A}\left(\frac{r_A}{1.2r_A}\right)^{1/2} = \frac{vr_B}{r_A\sqrt{1.2}}$$

26. $$\frac{1}{2}mv_{\min}^2 = \left[-\frac{GMm}{r} - \frac{GMm}{r}\right] - \left[-\frac{GMm}{(2r-a)} - \frac{GMm}{a}\right]$$

$$= \frac{2GMm(a^2 - 2ar + r^2)}{ar(2r-a)}$$

or $$v_{\min} = \sqrt{\frac{GM}{a}} \times \frac{2(r-a)}{[r(2r-a)]^{1/2}}$$

So, $$K = \frac{2(r-a)}{[r(2r-a)]^{1/2}}$$

27. $m_1r_1 = m_2r_2\ r_1 = \dfrac{m_2 r}{m_1 + m_2}$ (i)

$$m_1 r_1 \omega^2 = \frac{Gm_1m_2}{r^2}\ \omega^2 = \frac{G(m_1+m_2)}{r^3}$$

[From (i)] or $r = \left[\dfrac{G(m_1+m_2)}{\omega^2}\right]^{1/3}$

$(m_1 + m_2)^{1/3} = 2m_1 + m_2 = 8$

and $m_2 - m_1 = 6$ (given)

which gives $m_1 = 1$ and $m_2 = 7$ units

$$\frac{m_1}{m_2} = \frac{1}{7}$$

28. Interstellar velocity $v' = \sqrt{\dfrac{GM}{r}} = R\sqrt{\dfrac{g}{(R+h)}}$

$= \sqrt{v^2 - v_e^2}$

where v = projection velocity

$$\frac{R^2 g}{(r+h)} = v^2 - 2gR \text{ Solving } v^2 = \frac{23gR}{11}$$

29. For observer, $T' = \dfrac{2\pi}{\omega_S - \omega_E} = \dfrac{T_S T_E}{T_E - T_S}$

$= T_E$ (given) or, $T_E^2 = 2T_S T_E\ \ T_S = T_E/2$

30. Time period is minimum for the satellites with minimum radius of the orbit i.e. equal to the radius of the planet. Therefore.

$$\frac{GMm}{R^2} = \frac{mv^2}{R} \Rightarrow V = \sqrt{\frac{GM}{R}}$$

$$T_{\min} = \frac{2\pi R}{\sqrt{\frac{GM}{R}}} = \frac{2\pi R\sqrt{R}}{\sqrt{GM}}$$

using $M = \dfrac{4}{3}pR^3 \cdot \rho\ \ T_{\min} = \sqrt{\dfrac{3\pi}{G\rho}}$

Using values $T_{\min} = 3000$ s

[CHEMISTRY]

31. Change in internal energy does not depend upon both.

32. $n_1 = 4, n_2 = \frac{1}{2}$

$\gamma_1 = \frac{5}{3}$ (for monatomic gas)

$\gamma_2 = \frac{7}{5}$ (for diatomic gas)

$$\frac{n_1 + n_2}{\gamma - 1} = \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1}$$

$$\Rightarrow \frac{4 + 0.5}{\gamma - 1} = \frac{4}{\frac{5}{3} - 1} + \frac{1/2}{\frac{7}{5} - 1}$$

$$\Rightarrow \frac{4.5}{\gamma - 1} = \frac{12}{2} + \frac{5}{4} = \frac{29}{4}$$

$$\Rightarrow \frac{\gamma - 1}{4.5} = \frac{4}{29}$$

$$\Rightarrow \gamma - 1 = \frac{4 \times 4.5}{29} = 0.62$$

$$\Rightarrow \gamma = 1.62$$

33. As the process is adiabatic,

$\Delta U = -\Delta W = 1.46 \times 10^6$ J (i)

Also, $\Delta U = nC_v\,\Delta T = \frac{nf}{2} R\,\Delta T = 10^3 \times \frac{f}{2} \times 8.3 \times 7$ J (ii)

From (i) and (ii), we get $f = 5$ (diatomic).

34. $U_i = \frac{5}{2} RT_0 + \frac{3}{2} R\left(\frac{7}{3} T_0\right) = 6\,RT_0$

$U_f = \frac{5}{2} RT_f + \frac{3}{2} RT_f = 4\,RT_f$

As $U_i = U_f$

$\therefore\ T_f = \frac{3}{2} T_0$

35. From the first law of thermodynamics, we have

$Q = \Delta U + W$

For path iaf,

$50 = \Delta U + 20$

$\therefore\ \Delta U = U_f - U_i = 30$ cal

For path ibf,

$Q = \Delta U + W$

$\Rightarrow\ W = Q - \Delta U = 36 - 30 = 6$ cal

36. $W_{ab} = \Delta Q - \Delta U = nC_P\,dT - nC_V\,dT$ (at constant pressure)

$= n(C_P - C_V)dT$

$= nRdT$

$= 2 \times R \times (500 - 300) = 400R$

37. Heat is extracted from the source in path DA and AB is

$$\Delta Q = \frac{3}{2}R\left(\frac{p_0V_0}{R}\right)+\frac{5}{2}R\left(\frac{2p_0V_0}{R}\right)=\frac{13}{2}p_0V_0$$

38. At constant volume the total energy will be utilized in increasing the temperature of gas

i.e., $(\Delta Q)_v = \mu C_v \Delta T = \mu C_v(120-100) = 80$

$$\Rightarrow \mu C_v = \frac{80}{20} = 4\,\text{J/K}$$

This is the heat capacity of 5 mol gas.

39 $\Delta S = \frac{q_{rev}}{T} = \frac{-80\times 200}{273}$ = **−58.6 cal/K**

40. $\Delta S = \frac{q_{rev}}{T} = \frac{540\times 18}{373}$ = **26.06 cal/K-mol**

41. For isothermal expansion of an ideal gas,

$$\Delta S_{sys} = nR\ln\frac{V_2}{V_1} = 2\times 8.314\times 2.303\log\frac{10}{2} = \mathbf{26.77\ J/K}$$

42. For isothermal process, $\Delta S_1 = nR\ln\frac{P_1}{P_2}$

$$= 2\times 8.314\times 2.303\log\frac{1}{2} = \mathbf{-11.52\ J/K}$$

43. As the process is adiabatic and reversible, $q_{rev} = 0$ and hence,

$$\Delta S_{univ} = \mathbf{0}$$

44. Process is adiabatic but irreversible. For it, $q_{irr} = 0$ and hence, $q_{rev} \neq 0$. ΔS_{sys} may be determined by first calculating the final temperature of the system. For adiabatic irreversible process,

$$n\cdot C_V\cdot(T_2-T_1) = -P_{ext}(V_2-V_1) = -P_2\left(\frac{nRT_2}{P_2}-\frac{nRT_1}{P_1}\right)$$

or, $n\cdot\frac{3}{2}R(T_2-T_1) = -nR\left(T_2-T_1\cdot\frac{P_2}{P_1}\right)$

or, $\frac{3}{2}(T_2-400) = -(T_2-400\times\frac{1}{20})$

or, $T_2 = 256\ \text{K}$

Now, $\Delta S_{sys} = n\cdot C_p\ln\frac{T_2}{T_1} - n\cdot R\ln\frac{P_2}{P_1}$

$$= 1\times\frac{5}{2}\times 8.314\times 2.303\times\log\frac{256}{400} - 1\times 8.314\times 2.303\times\log\frac{1}{20}$$

$$= \mathbf{15.63\ J/K}$$

45. $\Delta S^\circ = \Sigma S^\circ_{Products} - \Sigma S^\circ_{Reactants}$

$= (2\times 256.23) - (2\times 248.53 + 1\times 205.03)$ = **−189.63 J/K**

46. $\Delta G = nRT\ln\frac{P_2}{P_1} = 2\times 8.314\times 373\times 2.303\times\log\frac{25}{10}$ = **5684.08 J**

47.

$$C(s)+\frac{1}{2}O_2(g)\longrightarrow CO(g)$$

$$\Delta n_g = \frac{1}{2}$$

$$\Delta H-\Delta U = \Delta n_g RT = \frac{1}{2}\times 8.314\times 298 = \mathbf{+1238.78\ J\ mol^{-1}}$$

48.

For isothermal reversible expansion $W = -2.303\, nRT \log \frac{P_1}{P_2}$

For all factors being same, $W \propto \frac{1}{\text{Molecular weight}}$

NO and **C_2H_6** both have equal molecular weights 30 g mol^{-1}.

49.

ΔH for isothermal free expansion is **zero.**

50.

$q = +\,40.65\ \text{kJ mol}^{-1}$

$W_{\text{exp.}} = -3.1\ \text{kJ}$

$\Delta E = q + W$

$= 40.65 - 3.1 = \mathbf{37.55\ kJ}$

51.

(I) $H_2O(l) \longrightarrow H^+(aq) + OH^-(aq)$; $\Delta H = 57.32\ \text{kJ}$

(II) $H_2(g) + \frac{1}{2}O_2(g) \longrightarrow H_2O(l)$; $\Delta H = -286.20\ \text{kJ}$

On adding I and II,

$H_2(g) + \frac{1}{2}O_2(g) \longrightarrow H^+(aq) + OH^-(aq)$;

$\Delta H = 57.32 - 286.2 = \mathbf{-\ 228.88\ kJ}$

52.

$C + O_2 \longrightarrow CO_2$; $\Delta H_1 = -\,393.5\ \text{kJ}$

$CO + \frac{1}{2}O_2 \longrightarrow CO_2$; $\Delta H_2 = -283\ \text{kJ}$

$\Delta H_2 = \Delta_f H_{CO_2} - \Delta_f H_{CO} - \frac{1}{2}\Delta_f H_{O_2}$

$\Delta_f H_{CO} = \Delta_f H_{CO_2} - \Delta H_2 - \frac{1}{2}\Delta_f H_{O_2}$

$= -\,393.5 - (-283) - 0 = -\ \mathbf{110.5\ kJ\ mol^{-1}}$

53.

Intrinsic energy means internal energy.

$\Delta H = \Delta E + \Delta n_g RT$

For, $C(s) + O_2(g) \longrightarrow CO_2(g)$

$\Delta H = -\,393\ \text{kJ}$,

$\Delta n_g = 1 - 1 = 0$, $T = 298\ \text{K}$

$-393 = \Delta E + 0 \Rightarrow \Delta E = -\ \mathbf{393\ kJ}$

54.

$\Delta H = \Delta E + \Delta n_g\, RT$

$C_2H_5OH(l) + 3O_2(g) \longrightarrow 2CO_2(g) + 3H_2O(l)$

$\Delta E = -1366.5 - (2-3) \times \frac{8.314}{1000} \times (27 + 273)$

$= -\,1366.5 + 2.4942 = -\ \mathbf{1364.0058\ kJ}$

55.

$$C_2H_5OH(l)+3O_2(g)\longrightarrow 2CO_2(g)+3H_2O(l)$$

$$\Delta U=\Delta H-\Delta n_g RT$$

$$=-1364.47-(2-3)\times\frac{8.314}{1000}\times(25+273)$$

$$=-1364.47+2.4776=-\mathbf{1361.99\ kJ\ mol^{-1}}$$

56. Heat of combustion is burning in excess of oxygen hence the reaction

$$C(s)+O_2\longrightarrow CO_2$$

58. Combustion of C is given by reaction $C(s)+O_2(g)\longrightarrow CO_2(g)$. Hence answer is (A)

59. Applying Hess's Law we get the value.

60. $C(s)+O_2\longrightarrow CO_2$; $\Delta H=-94$ Kcal mol^{-1} ...(1)

$H_2+\frac{1}{2}O_2\longrightarrow H_2O$; $\Delta H=-68$ Kcal mol^{-1} ...(2)

To get, $C(s)+2H_2(g)\longrightarrow CH_4(g)$

Multiple (2) with 2, add (1) & (2) and subtract (3)

$\Delta H=-94+2(-68)-(-213)=-17$ Kcal

[MATHEMATICS]

61. (b) Suppose that $\angle A=x^0$, then $\angle B=x+10^o$,

$\angle C=x+20^o$ and $\angle D=x+30^o$

So, we know that $\angle A+\angle B+\angle C+\angle D=2\pi$

Putting these values, we get

$(x^o)+(x^o+10^o)+(x^o+20^o)+(x^o+30^o)=360^o$

$\Rightarrow x=75^o$

Hence the angles of the quadrilateral are $75^o, 85^o, 95^o, 105^o$.

Trick : In these type of questions, students should satisfy the conditions through options. Here (b) satisfies both the conditions *i.e.* angles are in A.P. with common difference 10^o and sum of angles is 360^o.

62. (a) As given $\frac{n}{2}\{2a+(n-1)d\}=\frac{m}{2}\{a+(m-1)d\}$

$\Rightarrow 2a(m-n)+d(m^2-m-n^2+n)=0$

$\Rightarrow (m-n)\{2a+d(m+n-1)\}=0$

$\Rightarrow 2a+(m+n-1)d=0$, $(\because m\neq n)$

$\therefore S_{m+n}=\frac{m+n}{2}\{2a+(m+n-1)d\}=\frac{m+n}{2}\{0\}=0$.

63. (b) We have $a_1 = a_2 = a_3 = 1$ and $d_1 = 1, d_2 = 2, d_3 = 3$.

Therefore, $S_1 = \frac{n}{2}(n+1)$ ......(i)

$S_2 = \frac{n}{2}[2n]$ ......(ii)

$S_3 = \frac{n}{2}[3n-1]$ ......(iii)

Adding (i) and (iii),

$S_1 + S_3 = \frac{n}{2}[(n+1)+(3n-1)] = 2\left[\frac{n}{2}(2n)\right] = 2S_2$

Hence correct relation $S_1 + S_3 = 2S_2$.

64. (d) $\log_a x + 2\log_a x + ... + a\log_a x = \frac{a+1}{2}$

$\Rightarrow \log_a x(1+2+........+a) = \frac{a+1}{2}$

$\Rightarrow \log_a x.\frac{a(a+1)}{2} = \frac{a+1}{2} \Rightarrow x = a^{1/a}$.

65. (a) Here $a = 1, 2, 3, ..., m$; $d = 1, 3, 5, ..., 2m-1$ and $n = n$, then $S_1 + S_2 + ... + S_m = \frac{1}{2}mn(mn+1)$

$\left[\text{Using } S = \frac{m}{2}(a+l). \text{ Since } S_1, S_2, S_3, ...S_m \text{ form an A.P.}\right]$

66. (d) $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$

$\Rightarrow (a_1 + a_{24}) + (a_5 + a_{20}) + (a_{10} + a_{15}) = 225$

$\Rightarrow 3(a_1 + a_{24}) = 225 \Rightarrow a_1 + a_{24} = 75$

($\because$ In an A.P. the sum of the terms equidistant from the beginning and the end is same and is equal to the sum of first and last term)

$a_1 + a_2 + ... + a_{24} = \frac{24}{2}(a_1 + a_{24}) = 12 \times 75 = 900$.

67. (c) Let $a - d$, a, $a + d$ be the roots of the equation $x^3 - 12x^2 + 39x - 28 = 0$

Then $(a-d)+a+(a+d)=12$ and $(a-d)a(a+d)=28$

$\Rightarrow 3a = 12$ and $a(a^2 - d^2) = 28$

$\Rightarrow a = 4$ and $a(a^2 - d^2) = 28$

$\Rightarrow 16 - d^2 = 7 \Rightarrow d = \pm 3$.

68. (a) $a_1 = 1, a_2 = r, a_3 = r^2, ...$

$\therefore 4a_2 + 5a_3 = 4r + 5r^2$

To be its minimum $\frac{d}{dr}(4r + 5r^2) = 0 \Rightarrow r = \frac{-2}{5}$.

69. (d) Given that sum $S = \frac{a(r^n - 1)}{r - 1} = \frac{a(1 - r^n)}{1 - r}$ ......(i)

$P = a(ar)(ar^2).........(ar^{n-1}) = a^n r^{1+2+.........+(n-1)}$

$= a^n r^{(n-1)n/2}$ i.e., $P^2 = a^{2n} r^{n(n-1)}$ ......(ii)

and $R = \frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + ...$ upto n terms

$= \frac{1}{a}\left(1 + \frac{1}{r} + \frac{1}{r^2} + ...\text{upto } n \text{ terms}\right)$

$= \frac{\frac{1}{a}\left[\left(\frac{1}{r}\right)^n - 1\right]}{\left(\frac{1}{r} - 1\right)} \left(\because \frac{1}{r} > 1\right)$ if $r < 1$

$= \frac{(1 - r^n)}{ar^{n-1}(1 - r)}$ .... (iii)

Therefore , $\frac{S}{R} = \frac{a(1 - r^n)}{1 - r} \times \frac{ar^{n-1}(1 - r)}{(1 - r^n)} = a^2 r^{n-1}$

or $\left(\frac{S}{R}\right)^n = (a^2 r^{n-1})^n = a^{2n} r^{n(n-1)} = P^2$.

70. (c) Let there be $2n$ terms in the given G.P. with first term a and the common ratio r .

Then, $a\frac{(r^{2n} - 1)}{(r - 1)} = 5a\frac{(r^{2n} - 1)}{(r^2 - 1)} \Rightarrow r + 1 = 5 \Rightarrow r = 4$.

71. (a) Since $f(x + y) = f(x)f(y)$ for all $x, y \in N$, therefore for any $x \in N$

$f(x) = f(x - 1 + 1) = f(x - 1)f(1)$

$= f(x - 2)[f(1)]^2 = ... = [f(1)]^x$

$\Rightarrow f(x) = 3^x$, $(\because f(1) = 3)$

Now $\sum_{x=1}^{n} f(x) = 120 \Rightarrow \sum_{x=1}^{n} 3^x = 120$

$\Rightarrow \frac{3(3^n - 1)}{(3 - 1)} = 120 \Rightarrow n = 4$.

72. (c) Here $G = (ab)^{1/2}$ and $G_1 = ar^1, G_2 = ar^2, ...G_n = ar^n$

Therefore $G_1 . G_2 . G_3 .....G_n = a^n r^{1+2+...+n} = a^n r^{n(n+1)/2}$

But $ar^{n+1} = b \Rightarrow r = \left(\frac{b}{a}\right)^{1/(n+1)}$

Therefore, the required product is

$a^n \left(\frac{b}{a}\right)^{1/(n+1) \, . \, n(n+1)/2} = (ab)^{n/2} = \{(ab)^{1/2}\}^n = G^n$.

Note : It is a well known fact.

73. (c) Since $\alpha, \beta, \gamma, \delta$ form an increasing G.P., so $\alpha\delta = \beta\gamma$ where $\alpha < \beta < \gamma < \delta$.

On solving $x^2 - 3x + a = 0$,

we get $x = \frac{1}{2}(3 \pm \sqrt{9-4a})$.

Also $\alpha < \beta$.

Hence $\alpha = \frac{1}{2}(3 - \sqrt{9-4a}),\ \beta = \frac{1}{2}(3 + \sqrt{9-4a})$

Similarly from $x^2 - 12x + b = 0$, we get

$\gamma = \frac{1}{2}(12 - \sqrt{144-4b}),\ \delta = \frac{1}{2}(12 + \sqrt{144-4b})$

74. (d) $1 - \cos\alpha = \frac{1}{2-\sqrt{2}} = 1 + \frac{1}{\sqrt{2}}$

$\Rightarrow \cos\alpha = -\frac{1}{\sqrt{2}} = \cos\frac{3\pi}{4} \Rightarrow \alpha = \frac{3\pi}{4}$.

75. (b) $5 = \frac{x}{1-r}$

$\Rightarrow 5 - 5r = x \Rightarrow r = 1 - \frac{x}{5}$

As $|r| < 1$ *i.e.,* $\left|1 - \frac{x}{5}\right| < 1 \quad -1 < 1 - \frac{x}{5} < 1$

$-5 < 5 - x < 5 = -10 < -x < 0 = 10 > x > 0$

i.e, $0 < x < 10$.

76. (b) Since A.M. > G.M., we have

$\frac{a+b}{2} > \sqrt{ab},\ \frac{b+c}{2} > \sqrt{bc}$ and $\frac{a+c}{2} > \sqrt{ac}$.

Multiplying these inequalities, we get

$(a+b)(b+c)(c+a) > 8abc$.

77. (b) As given $b^2 = ac$ and

$2(\log 2b - \log 3c) = \log a - \log 2b + \log 3c - \log a$

$\Rightarrow b^2 = ac$ and $2b = 3c \Rightarrow b = 2a/3$ and $c = 4a/9$

Since $a + b = \frac{5a}{3} > c,\ b + c = \frac{10a}{9}, > a,\ c + a = \frac{13a}{9} > b$

It implies that a, b, c form a triangle with a as the greatest side.

Now, let us find the greatest angle A of ΔABC by using the cosine formula.

$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = -\frac{29}{48} < 0$

$\therefore$ The angle A is obtuse.

78. (a) Let the two quantities be a and b. Then a, A_1, A_2, b are in A.P.

$\therefore A_1 - a = b - A_2 \Rightarrow A_1 + A_2 = a + b$ ......(i)

Again a, G_1, G_2, b are in G.P.

$\therefore \frac{G_1}{a} = \frac{b}{G_2} \Rightarrow G_1G_2 = ab$ .....(ii)

Also a, H_1, H_2, b are in H.P.

$\therefore \frac{1}{H_1} - \frac{1}{a} = \frac{1}{b} - \frac{1}{H_2} \Rightarrow \frac{1}{H_1} + \frac{1}{H_2} = \frac{1}{a} + \frac{1}{b}$

$\Rightarrow \frac{H_1 + H_2}{H_1H_2} = \frac{a+b}{ab} = \frac{A_1 + A_2}{G_1G_2}$ [By (i) and (ii)]

$\therefore \frac{G_1G_2}{H_1H_2} = \frac{A_1 + A_2}{H_1 + H_2}$.

79. (a) Let numbers be x and y.

Then $A = \frac{1}{2}(x+y), \sqrt{xy} = G$ or $G^2 = xy$

and $\frac{2xy}{(x+y)} = 4$,

$\Rightarrow G^2 = 4A$

Also, $2A + G^2 = 2A + 4A = 27 \Rightarrow A = \frac{9}{2}$

So, $x + y = 9, xy = 18$

Hence numbers are 6, 3.

80. (b) As we know $A.M. > G.M. > H.M.$

$\therefore A.M. = 15, G.M. = 12, H.M. = \frac{144}{15}$.

81. (b) **Trick :** Put $b = 1$ and $c = 8$ so that $a = 4.5$ and $G_1 = 2, G_2 = 4$. Now $G_1^3 + G_2^3 = 72$.

Also option (b) gives this value *i.e.* $2 \times 2 \times 4 \times \frac{9}{2} = 72$.

82. (c) a, ar, ar^2 are in G.P. $a, ar-8, ar^2-64$ are in A.P., we get $\Rightarrow a(r^2-2r+1)=64$ .....(i)

Again, $a, ar-8, ar^2-64$ are in G.P.

$\therefore (ar-8)^2 = a(ar^2-64)$ or $a(16r-64)=64$ .....(ii)

Solving (i) and (ii), we get $r=5, a=4$. Thus required numbers are $4, 20, 100$.

Trick : Check by alternates according to conditions

(a) $\Rightarrow$ 4, 20, – 28 which are not in A.P.

(b) $\Rightarrow$ 4, 12, – 28 which are also not in A.P.

(c) $\Rightarrow$ 4, 20, 36 which are obviously in A.P. with 16 as common difference. These numbers also satisfy the second condition *i.e.* 4, 20 – 8, 36 are in G.P.

83. (b) x, y, z are in G.P. Hence $y^2 = xz$

$\therefore 2\log y = \log x + \log z$

$\Rightarrow 2(\log y + 1) = (1+\log x) + (1+\log z)$

$\Rightarrow 1+\log x, 1+\log y, 1+\log z$ are in A.P.

$\Rightarrow \frac{1}{1+\log x}, \frac{1}{1+\log y}, \frac{1}{1+\log z}$ are is H.P.

84. (d) $\frac{1}{2}\left[2^{\sin\theta} + 2^{\cos\theta}\right] \geq \sqrt{2^{\sin\theta}2^{\cos\theta}}$ ($\because$ A.M. $\geq$ G.M.)

$\Rightarrow 2^{\sin\theta} + 2^{\cos\theta} \geq 2.2^{(\sin\theta+\cos\theta)/2}$ .....(i)

Now $(\sin\theta + \cos\theta) = \sqrt{2}\sin(\theta + \pi/4) \geq -\sqrt{2}$

for all real θ

$2^{\sin\theta} + 2^{\cos\theta} \geq 2.2^{(\sin\theta+\cos\theta)/2} > 2.2^{-\sqrt{2}/2}$

$\Rightarrow 2^{\sin\theta} + 2^{\cos\theta} \geq 2^{1-(1/\sqrt{2})}$.

85. (a) $\frac{(a+b)+(c+d)}{2} \geq \sqrt{(a+b)(c+d)}$

or $\frac{2}{2} > \sqrt{M}$ or $1 \geq M$

Also $M > 0$. So, $0 < M \leq 1$.

86. (d) $b = a + d, c = a + 2d$, where $d > 0$

Now $a^2, (a+d)^2, (a+2d)^2$ are in G.P.

$\therefore (a+d)^4 = a^2(a+2d)^2$

or $(a+d)^2 = \pm a(a+2d)$

or $a^2 + d^2 + 2ad = \pm (a^2 + 2ad)$

Taking (+) sign, $d = 0$ (not possible as $a < b < c$)

Taking (–) sign,

$2a^2 + 4ad + d^2 = 0$, $\left[\because a + b + c = \frac{3}{2}, \ \therefore a + d = \frac{1}{2}\right]$

$2a^2 + 4a\left(\frac{1}{2} - a\right) + \left(\frac{1}{2} - a\right)^2 = 0$ or $4a^2 - 4a - 1 = 0$

$\therefore a = \frac{1}{2} \pm \frac{1}{\sqrt{2}}$.Here $d = \frac{1}{2} - a > 0$. So, $a < \frac{1}{2}$.

Hence $a = \frac{1}{2} - \frac{1}{\sqrt{2}}$.

87. (c) This series is clearly an A.G., the corresponding A.P. is $1 + 4 + 7 + 10 + ...$ having n^{th} term $= 3n - 2$ and corresponding G.P. is $1 + \frac{1}{5} + \frac{1}{5^2} + ...$ having n^{th} term $= \frac{1}{5^{n-1}}$

Hence required n^{th} term of the series is $\frac{3n-2}{5^{n-1}}$.

Trick : Check by putting $n = 1, 2$ in alternates.

88. (b) Let T_n be the n^{th} term of the series

$$\frac{1}{1+1^2+1^4} + \frac{2}{1+2^2+2^4} + \frac{3}{1+3^2+3^4} + ...$$

Then $T_n = \frac{n}{1+n^2+n^4} = \frac{n}{(1+n^2)^2 - n^2}$

$$= \frac{n}{(n^2+n+1)(n^2-n+1)}$$

$$= \frac{1}{2}\left[\frac{1}{n^2-n+1} - \frac{1}{n^2+n+1}\right]$$

$$= \frac{1}{2}\left[\frac{1}{1+(n-1)n} - \frac{1}{1+n(n+1)}\right]$$

Now $\sum_{r=1}^{n} T_r = \frac{1}{2}\left[\frac{1}{1} - \frac{1}{1+1.2}\right] + \frac{1}{2}\left[\frac{1}{1+1.2} - \frac{1}{1+2.3}\right]$

$+\frac{1}{2}\left[\frac{1}{1+2.3} - \frac{1}{1+3.4}\right] + ... + \frac{1}{2}\left[\frac{1}{1+(n-1)n} - \frac{1}{1+n(n+1)}\right]$

$= \frac{1}{2}\left[1 - \frac{1}{1+n(n+1)}\right] = \frac{n(n+1)}{2(n^2+n+1)}$.

Trick : Checking for $n = 1, 2$. $S_1 = \frac{1}{3}$ and $S_2 = \frac{3}{7}$

which are given by (b).

89. (c) Obviously $T_n = \frac{1^3 + 2^3 + 3^3 + ... + n^3}{1 + 3 + 5 + ...\text{upto } n \text{ terms}}$

$= \frac{\Sigma n^3}{\frac{n}{2}[2 + (n-1)2]} = \frac{1}{4}\frac{n^2(n+1)^2}{n^2} = \frac{1}{4}(n^2 + 2n + 1)$.

90. (d) $\frac{1}{\sqrt{2}+\sqrt{1}} + \frac{1}{\sqrt{3}+\sqrt{2}} + \frac{1}{\sqrt{4}+\sqrt{3}} + ... + \frac{1}{\sqrt{n^2}+\sqrt{n^2-1}}$

Rationalization of D^r

$\therefore S = (\sqrt{2} - \sqrt{1}) + \left(\sqrt{3} - \sqrt{2}\right) + ... + \left(\sqrt{n^2} - \sqrt{n^2 - 1}\right)$

$S = n - 1$.